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Research Article

Numerical simulation of the synthetic strain energy and crack characterization parameters using the FEM method of a two-dimensional multi-position model

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Fracture mechanics is a science that studies the growth and propagation of cracks, as well as the ability to absorb cracks of a component or material under service conditions (operation, service life etc.). This paper deals with the numerical modeling of the strain energy evolution (ALLAE), the J integral and stress intensity factors, of a multi-position initial crack of length $a = 1\text{mm}$. The first part is based on the study of the positions of the cracks of the upper face which contain positive values, and the second part of the study is based on the study of the positions of the cracks of the lower face which contain negative values. The finite element method was used on a two dimensional model in the first mode I. Additionally, elasto-plastics material was applied. Thus, the CPS8, 8-node biquadratic plane stress quadrilateral elements were used. The crack is then modeled numerically using the ABAQUS finite element calculation code. In addition, the results obtained concerning the numerical modeling were compared, and discussed between the different positions: either higher dimensions $y=8, 6.4, 4.8, 3.2$ and 1.6mm or lower dimensions $y=-6.4, -4.8, -3.2$ and -1.6mm . A good correspondence was obtained between the different comparison results in all the modeling cases of our work. When there is a crack on the upper face, the real KI varies between 50 and 92 ($\text{Mpa}\sqrt{\text{m}}$), the KII varies between -8 and 8.8 ($\text{Mpa}\sqrt{\text{m}}$). Thus, the integral J varies between 1×10^{-8} and 1.2×10^{-7} (KJ/m^2) and the dissipation energy ALLAE varies between 0 and 3×10^{-11} (J). In addition, when there is a crack on the lower side, the varied KI factor between 45 and 8 ($\text{Mpa}\sqrt{\text{m}}$), KII varies between -0.5 and 7 ($\text{Mpa}\sqrt{\text{m}}$) Thus, the integral-J varies between 3×10^{-8} and 1×10^{-7} (KJ/m^2) and the dissipation energy ALLAE varies between 0 and 2×10^{-11} (J).

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1. Introduction

Fracture mechanics or crack mechanics is the field of mechanics that studies the formation of cracks in materials. It also depends on the basis of the numerical simulation of the different characterization parameters at the crack front, among which the artificial deformation energies, the stress intensity factors and the contour integral J. Therefore, many of the recent works published in this field among.

Gozin and Aghaie-Khafri [1] presented a study based on the design of plasticity induced crack closure (PICC) by FEM method to study the effect of the stress, compressive residual field on crack fatigue growth. Yazhe et al [2] used local radial base point interpolation method (LRPIM) and combined by elasto-plastic theory to analyze crack propagation in

elasto-plastic materials. Zizheng Sun et al [3] presented numerical simulations with CEM regarding several benchmark tests, the results of which further indicate the ability of CEM to capture complex crack growths. Numerical methods have been widely used to calculate the fracture parameters, in particular in the linear elastic plastic fracture mechanics presented by Bouchard et al [4], the dynamic fracture mechanics Réthoré et al [5], fatigue (Miranda et al [6]) and quasi-static crack propagation Khoei et al [7]. Thus, Azocar et al [8] proposed a new method for the simulation of crack propagation in solids (LEFM) in (2D). On the other hand, a new method stretching finite element method (SFEM) has been used by Bentahar et al [9], Bentahar and Benzaama [10] to characterize the stress intensity factors of an initial crack.

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For the comparative analysis Kim et al [11] proposed a new numerical simulation methodology, by the finite element method (FEM) to predict the fatigue life of tires and calculated the cracking energy density (CED), and the virtual crack closure technique (VCCT) was used to calculate the strain energy release rate (SERR) of a crack. Barhli et al [12] analyzed the measured displacement fields, using a finite element method to extract the elastic strain energy release rate as an integral J. Bentahar et al [13] used the 2D extended finite element method (X-FEM) in mode I, to model the propagation of the crack and evaluated the energy (ALLSE).

2. Crack Parameters

2.1. Stress intensity factor

Saverio [14] defined the stress intensity factor K, which is the only significant parameter, which allows to know the state of stress and strain at any crack point.

$$KI = F\sigma\sqrt{a\pi} \quad (1)$$

σ is the applied stress;

a is the crack length.

Where, F is the geometric correction factor of the model used.

Where, F is the geometric correction factor of the model used.

$$F = 1.12 - 0.23(a/w) + 10.6(a/w)^2 - 21.7(a/w)^3 + 30.4(a/w)^4 \quad (2)$$

Where the stress intensity factor KII is calculated by the relation.

$$K_I \sin \theta + K_{II} (3 \cos \theta - 1) = 0 \quad (3)$$

2.2. Contour integral - J

Several authors in fracture mechanics have allowed to model the problem of the presence of a crack in an in-depth way and have developed the calculation methods.

Among these authors Rice [15] and Bui [16] with contour integrals (J), Nguyen [17] and Destuynder [18] by introducing an arbitrary field in the formulation of the integral they have approached. Indeed, work has

been developed on the basis of elasticity in small displacements and mainly addresses the first phase of the cracking process.

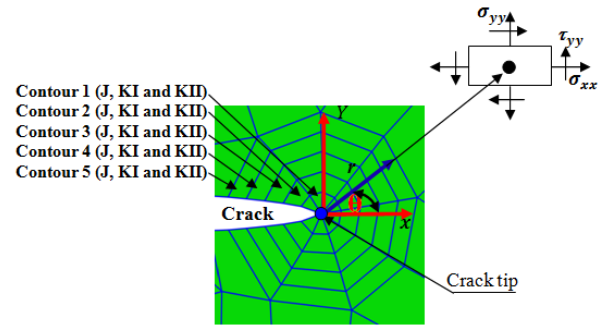


Figure 1. Stress field and the different contours in the vicinity of the crack front.

Tada et al [19] gave the general 2D stress field equation near the crack tip to define the stress intensity factor.

$$\sigma_{i,j}^{I,II} (r, \theta) = \frac{K_{I,II}}{\sqrt{2\pi r}} f_{ij}(\theta) \quad (4)$$

$\sigma_{i,j}^{I,II}$ is the stress field associated with mode I. KI, II is the SIF in mode I and II,

$$\begin{aligned} \sigma_{xx} &= \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) \\ \sigma_{yy} &= \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) \\ \tau_{xy} &= \frac{K_I}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2} \end{aligned} \quad (5)$$

3. Numerical simulation model

Our model was made up of 4636 elements and 4780 nodes, the material characteristic is: The Young's modulus $E=10^7$ Pa and Poisson ratio is $\nu=0.25$ the dimensions of the plate are: $L=16$ mm and $C=8$ mm. The crack positions relative to the crack in the middle signified by $y= 6.4, 4.8, 3.2,$ and 1.6 mm from upper side and from lower side $y= -6.4, -4.8, -3.2,$ and -1.6 mm see figure 2. 8-node CPS8 biquadratic plane stress quadrilateral elements were used for the numerical simulation. This type of element is well suited to simulation. In the same way, one uses singular

elements around the front of crack. The types of these "quarter point" singular elements are reduced quadratic elements. The mesh around the singularity area is refined based on the contour number.

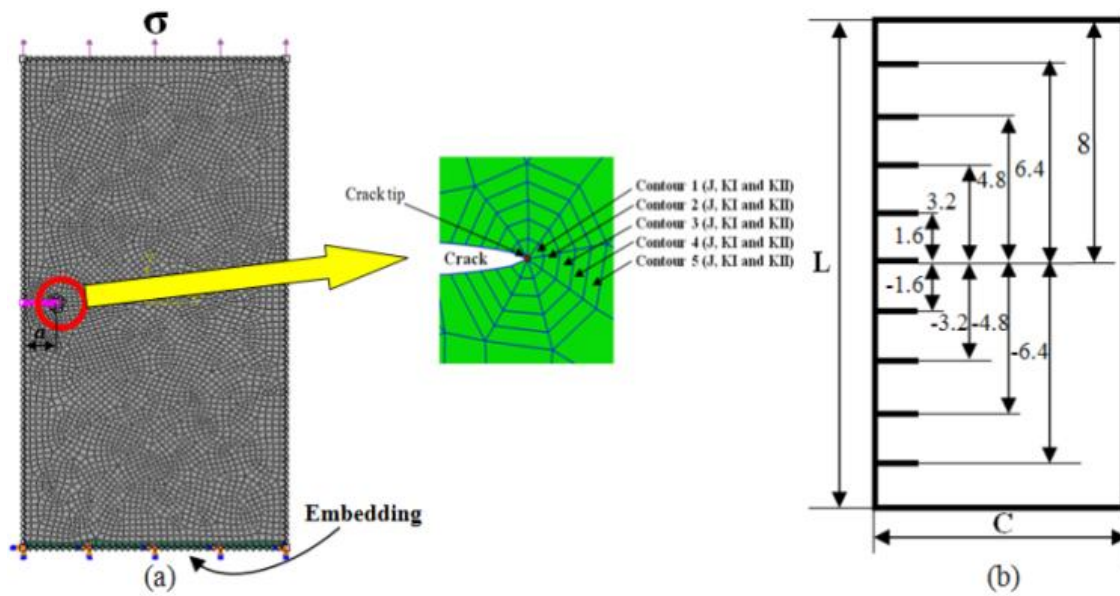


Figure 2. FEM model: a) boundary conditions with the crack front; b) the dimensions of the study model with different crack positions.

4. Results and discussion

4.1. Top side initial crack

We present in this paragraph a structure, which contains

a crack positioned on the upper side above the middle, of the structure studied see figure 4. The different dimensions of the upper side give by $y=6.4, 4.8, 3.2,$ and 1.6mm .

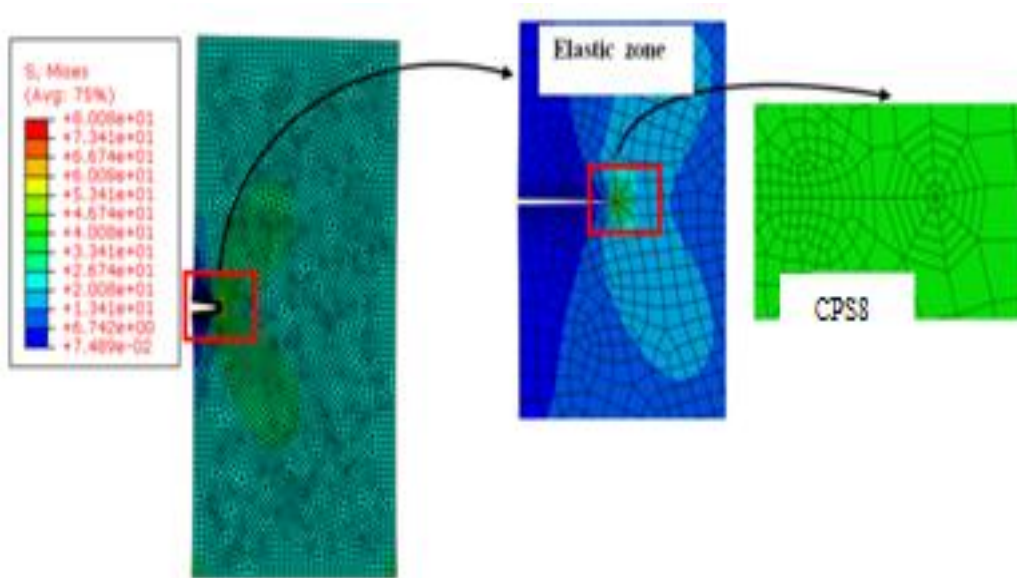


Figure 3.FEM model with plastic zone and CPS8 elements.

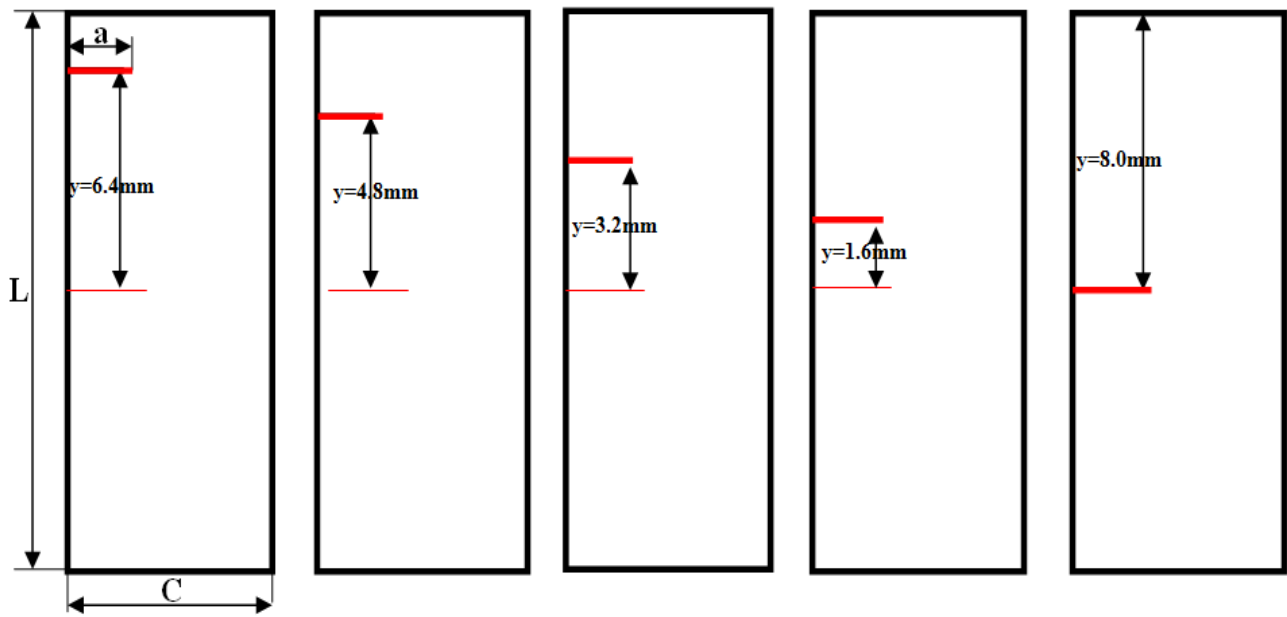


Figure 4. 2D model of upper side regarding the different dimensions the (y).

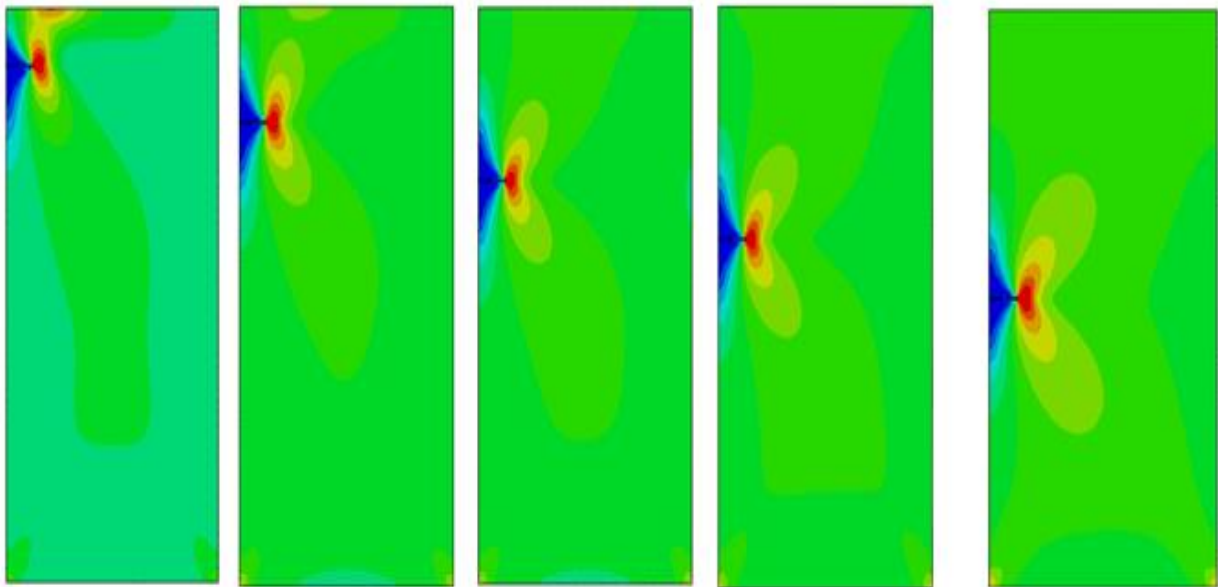


Figure 5. 2D model of upper side with plastic zone presentation.

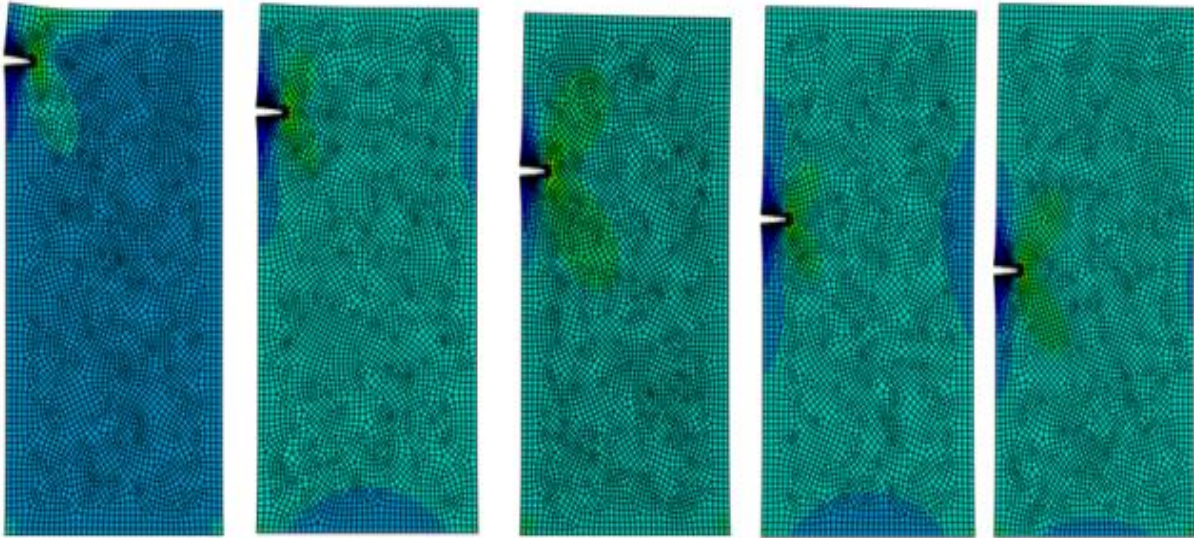


Figure 6. Top side 2D FEM model regarding different crack positions.

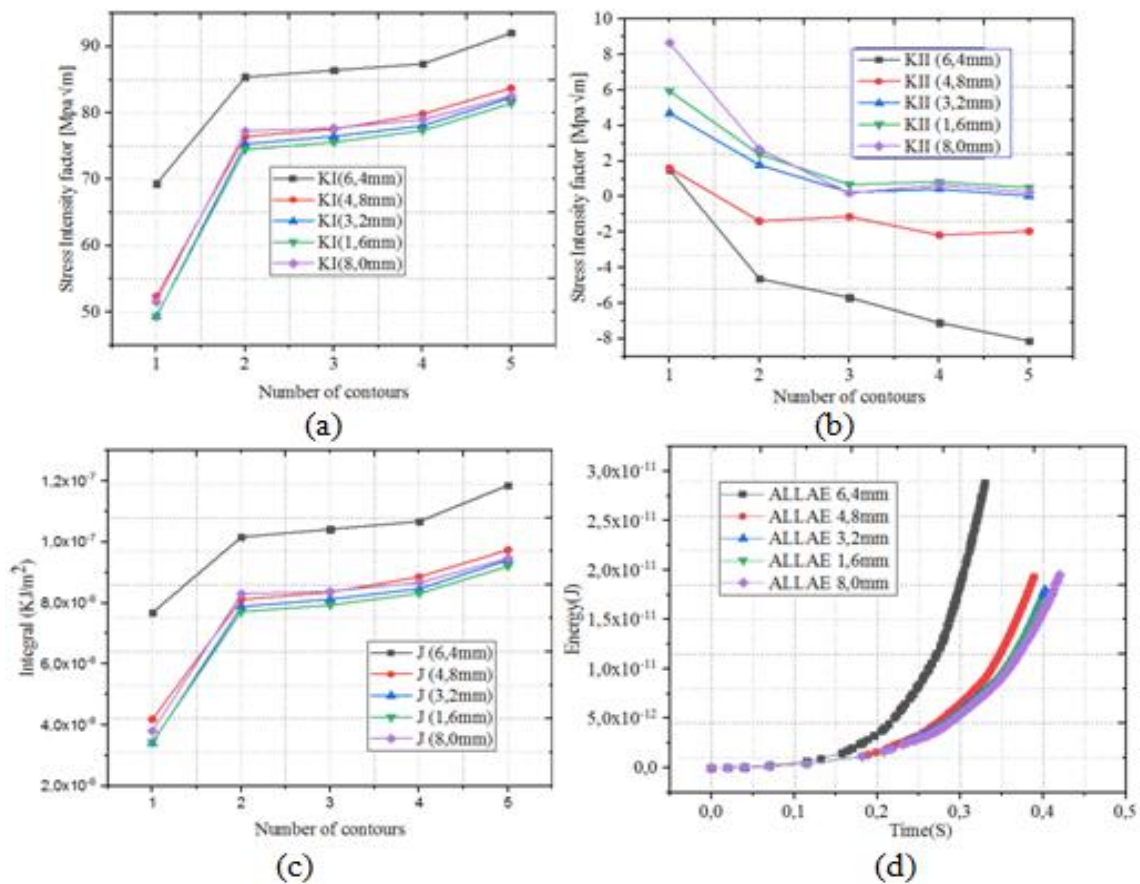


Figure 7. Evolution of the upper side (ALLAE) energy concerning the values -1.6, -3.2, -4.8, -6.4 and 8mm.

Figure 7 shows the evolution of the stress intensity factor and the J-integral as a function of the contour number, fig7(c) presents the relationship between ALLAE energy as a function of time, the evolutions of this study pose the problem of the position of the crack at the upper side of the structure. Over time we can see that the ALLAE dissipation energy increases, especially after 0.15s. This evolution shows a good

correlation between the different crack dimensions of $y = 6.4, 4.8, 3.2, 1.6$ and 8mm . There is a very good correspondence between the results obtained in the case of $y = 4.8, 3.2, 1.6\text{mm}$ and the reference value of $y = 8\text{mm}$ in all cases of comparisons except the value of $y = 6.4\text{mm}$, because this space is the longest, even for the energy results (ALLAE). In the case where the crack is larger, the factor (KI) varies between 50 and

92 MPa, and KII varies between -8 and 8.8 MPa, so the J-integral varies between 3×10^{-8} and 1.2×10^{-7} KJ/ m² and the dissipation energy (ALLAE) varies from 0 to 3×10^{-11} J.

We propose in this part a structure which contains an initial crack positioned on the lower side below the crack which is located in the middle, see figure 8. The different dimensions of the lower side given by $y=-6.4$, -4.8, -3.2, and -1.6mm.

4.2. Bottom side crack

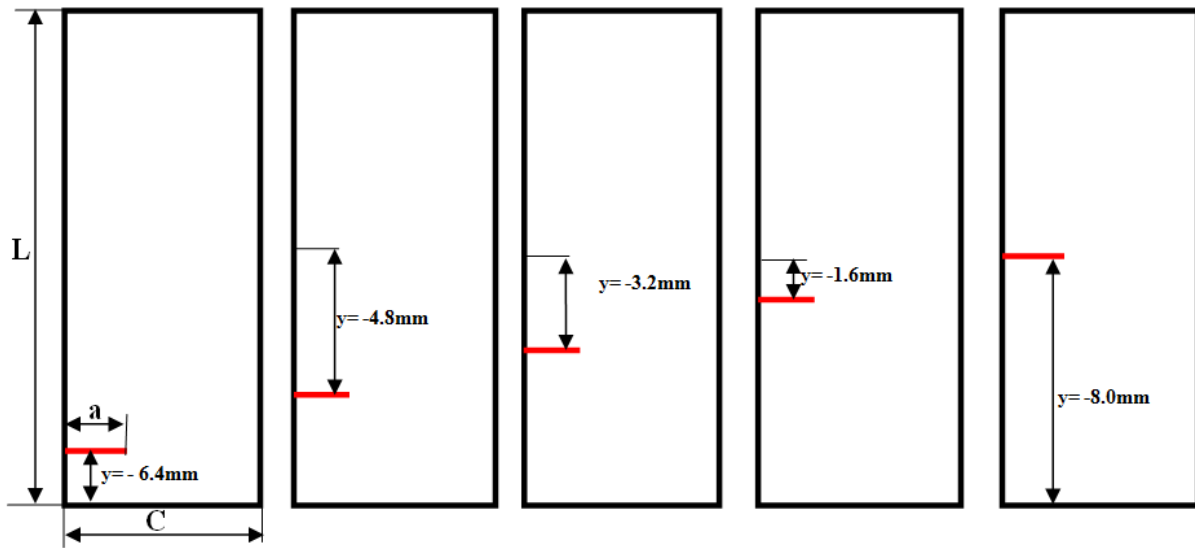


Figure 8. 2D model of lower side presenting dimensions of (y).

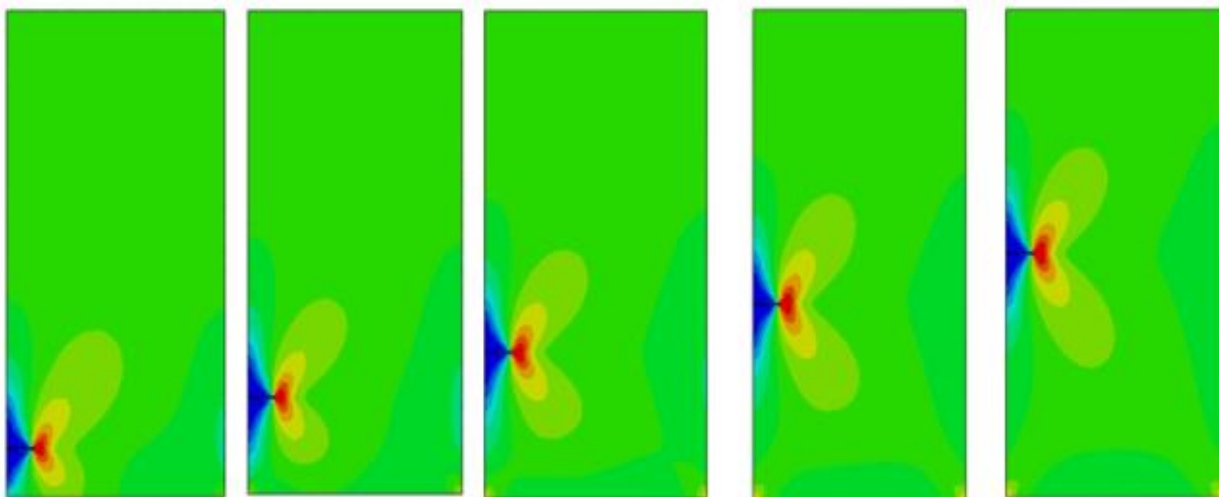


Figure9. Illustration of the Upper Side 2D Model with the Plastic Zone.

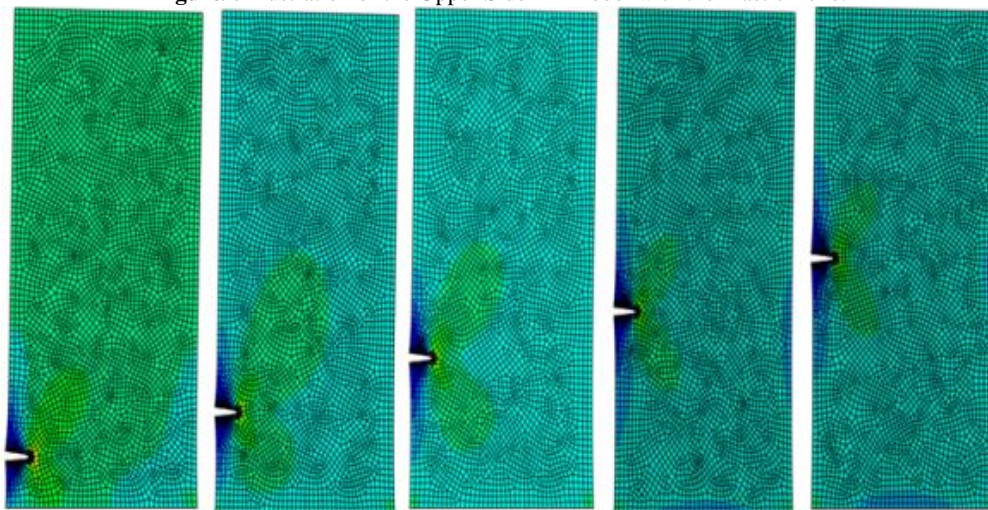


Figure 10. Lower side 2D FEM model for values -1.6, -3.2, -4.8, -6.4 and 8mm.

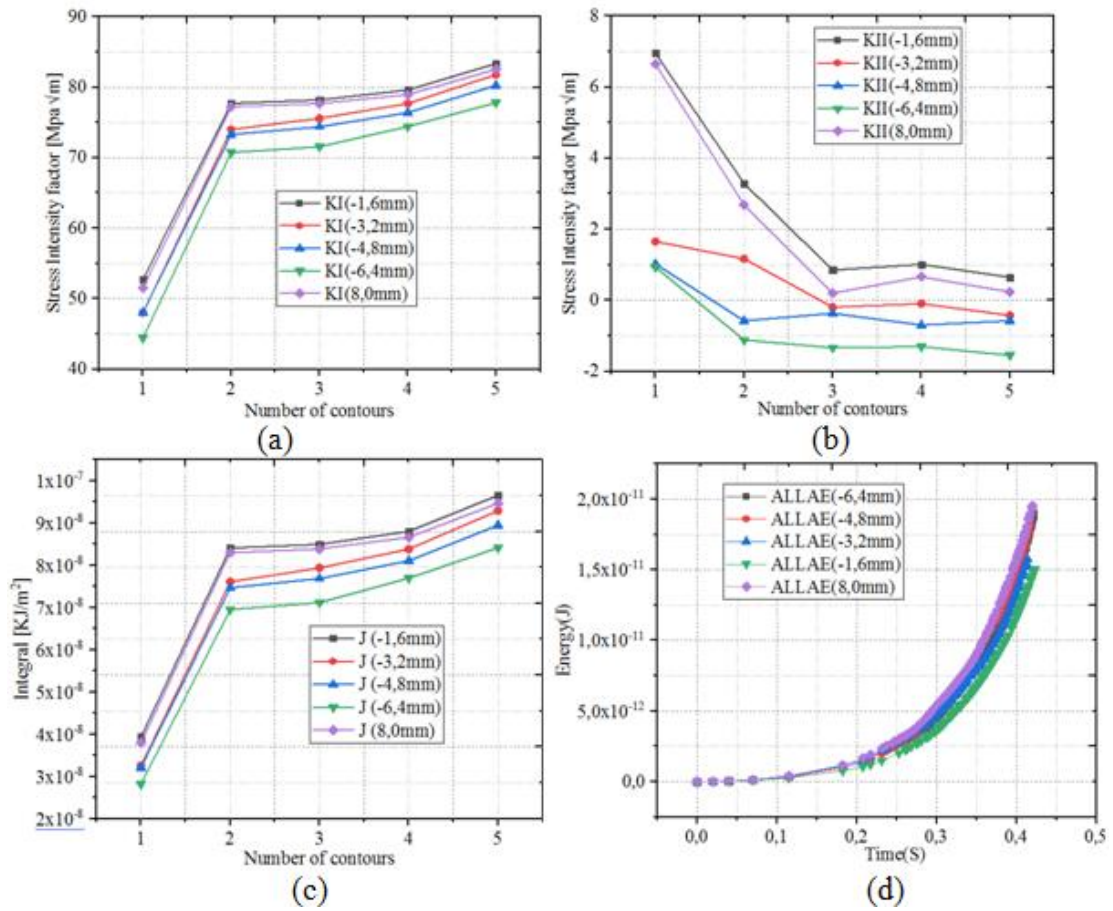


Figure 11. Evaluation of different crack parameters at the front; a) KI, b) KII, c) Integral J and d) Energy ALLAE.

Figure 11 explains the evolution of different crack parameters to know the states at the crack front it is necessary to characterize the different parameters.

The evaluation of different crack parameters such as stress intensity factors, contour integral J and ALLAE energy are shown in Fig. 11. We can see that the increase in ALLAE dissipation energy in this case

starts after the time 0.11s, the bottom side crack needs energy early. In the cases where the crack is higher the factor KI, true between 45 and 85Mpa√m, KII varies between -0.5 and 7 Mpa√m . Similarly, the J- integral varies between 3×10⁻⁸ and 1×10⁻⁷KJ/m² and the dissipation energy ALLAE varies between 0 and 2×10⁻¹¹J.

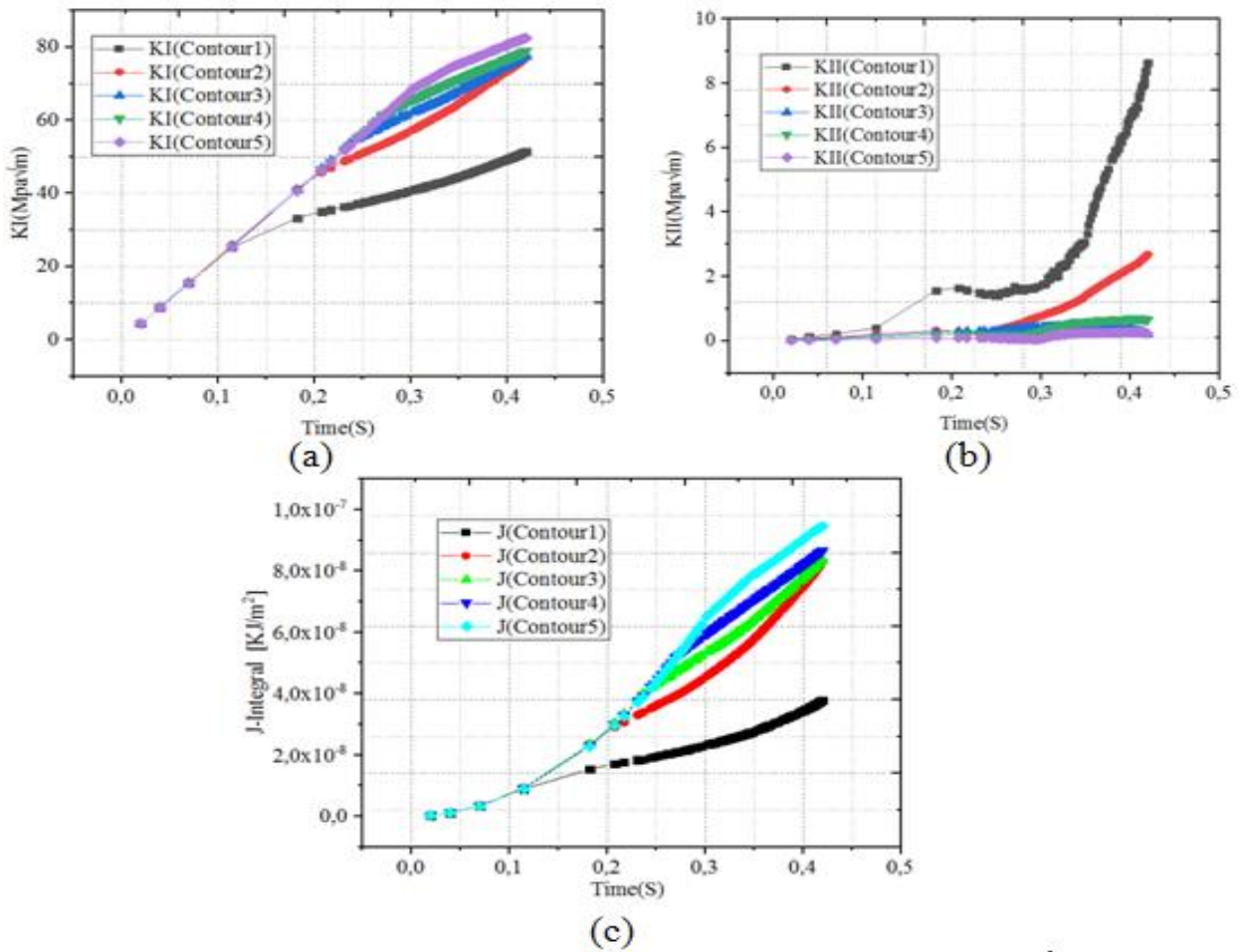


Figure 12. Evaluation of different crack parameters at the front; a) KI, b) KII and c) Integral J.

The evaluation of the stress intensity factors, and the integral of the contour J as a function of time, are illustrated in figure 12, we note that the more time increases the more the different crack parameters

increase, the intensity factor of contour KI and J-integral of the contour as a function of time increases with each increase in number of the contour and conversely for KII.

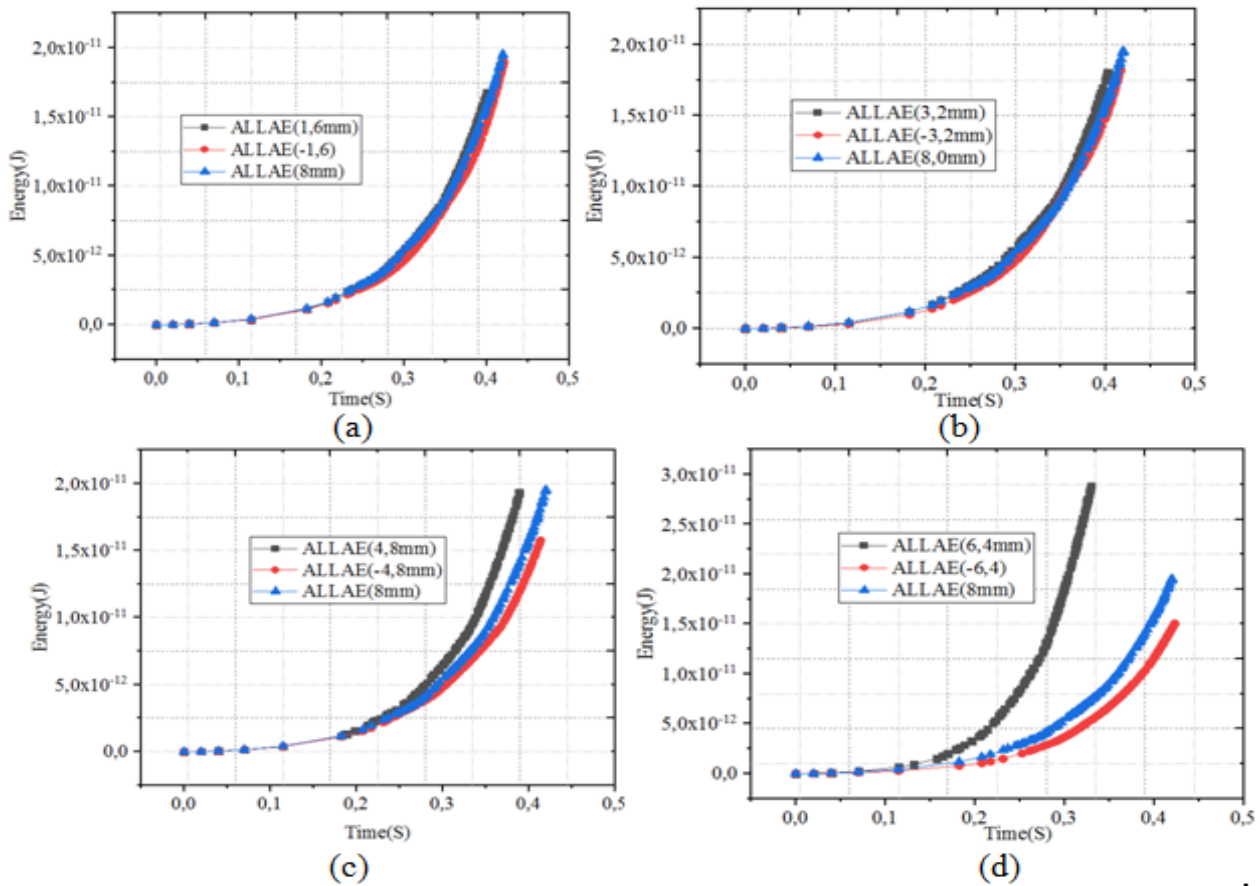


Figure 13: Comparison of ALLAE energy for different values of y ; a) $y=1.6$ and 1.6mm , b) $y=-3.2$ and 3.2mm , c) $y=-4.8$ and 4.8mm and d) $y=-6.4$ and 6.4mm

Figure 13 shows a comparison of strain energy (ALLAE), for different values of (y) relative to the initial crack in the middle (lower and upper). It can be seen that the greater the distance between the crack in the middle and the other cracks, at the bottom and at the top, the greater the dispersion of the energy (ALLAE). Thus, one can say that in the event of convergence of crack, the energy is well preserved.

5. Conclusion

Two cases of numerical simulation were studied, on the one hand a crack located on the upper face of the model by positive dimensions at top, the other study based on a crack located on the lower face of the model at the bottom.

Numerical simulation by the finite element method was used to characterize the different crack parameters of a two-dimensional multi-position model.

The position of the crack plays an important role on the influence of the results obtained in our work. In the same way justified, that the position of crack influences in an important way on the parameters of the crack. Thus, we obtained the low values concerning the integral of the contour J and in the energy (ALLAE). On the other hand, the values of the stress intensity factor results (KI and KII)

in mode I are higher. There is a proportionality between the results obtained from the SIF, the contour J -integral as a function of contour number and the energy (ALLAE) as a function of time. The study of the energy at the level of the crack front has been done by Bentahar et al. [13] in the case of the strain energy; by Bentahar [20] in the case of the analysis of the energy dissipation by the XFEM method and in the case of the analysis of the energy dissipation; and by Bentahar [21] in the case of fatigue analysis of an inclined crack propagation.

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Author contributions

Conceptualization: [Mohammed Bentahar, Habib Benzaama], Methodology: [Mohammed Bentahar, Habib Benzaama], Formal analysis and investigation: [Mohammed Bentahar, Habib Benzaama], Writing - original draft preparation: [Mohammed Bentahar, Habib Benzaama]; Writing - review and editing: [Mohammed Bentahar, Habib Benzaama], Funding acquisition: [Mohammed Bentahar, Habib Benzaama], Resources:

[Mohammed Bentahar, Habib Benzaama],Supervision:

[Mohammed Bentahar, Habib Benzaama]

Conflicts of interest/Competing interests

The authors declare that they have no conflict of interest in the work submitted for publication.

References

- [1] M.H. Gozin and M. Aghaie-Khafri, 2012, "2D and 3D finite element analysis of crack growth under compressive residual stress field", International Journal of Solids and Structures, Vol. 49, (23–24), 15 November, pp. 3316-3322, <https://doi.org/10.1016/j.ijsolstr.2012.07.014>
- [2] L. Yazhe, X. Nengxiong, T. Jinzhi and M. Gang, 2019, "Comparative modelling of crack propagation in elastic-plastic materials using the meshfree local radial basis point interpolation method and eXtended finite-element method", 6(11) <https://doi.org/10.1098/rsos.190543>.
- [3] Z. Sun, X. Zhuang and Y. Zhang, 2019, "Cracking Elements Method for Simulating Complex Crack Growth", J. Appl. Comput. Mech, 5(3) pp. 552-562 DOI: [10.22055/JACM.2018.27589.1418](https://doi.org/10.22055/JACM.2018.27589.1418).
- [4] P.O. Bouchard, F. Bay and Y. Chastel, 2003, "Numerical modelling of crack propagation automatic remeshing and comparison of different criteria, Computer Methods Applied Mechanics Engineering, 192 (35/36), pp.3887-3908.
- [5] J. Réthoré, A. Gravouil and A. Combescure, 2005, "An energy-conserving scheme for dynamic crack growth using the extended finite element method", International Journal for Numerical Methods in Engineering, 63(5), pp. 631-659.
- [6] A.C.O. Miranda, M.A. Meggiolaro, J.T.P. Castro, L.F. Martha and T.N. Bittencourt 2003, "Fatigue life and crack predictions in generic 2D structural components", Engineering Fracture Mechanics, 70 (10), pp.1259-1279.
- [7] A.R. Khoei, H. Azadia, and H. Moslemia, 2008, "Modeling of crack propagation via an automatic adaptive mesh refinement based on modified superconvergent patch recovery technique", Engineering Fracture Mechanics, 75, pp.2921-2945.
- [8] D. Azocar, M. Elgueta, and M.C. Rivara, 2010, "Automatic LEM crack propagation method based on local Lepp-Delaunay mesh refinement", Advances in Engineering Software, 41, pp. 111-119.
- [9] M. Bentahar, H. Benzaama, M. Bentoumi and M. Mouktari, A new automated stretching finite element method for 2D crack propagation, Journal of Theoretical and Applied Mechanics (JTAM), Vol. 55, No. 3, pp. 869-881, 2017, [https://DOI:10.15632/jtam-pl.55.3.869](https://doi.org/10.15632/jtam-pl.55.3.869).
- [10] M. Bentahar, H. Benzaama, Numerical Simulation of 2D Crack Propagation using SFEM Method by Abaqus, Tribology and Materials, vol. 1, No. 4, 2022, pp. 145-149; <https://doi.org/10.46793/tribomat.2022.018>
- [11] T.W. Kim, H.Y. Jeon, and J.H. Choe, 2005, "Prediction of The Fatigue Life of Tires Using CED and VCCT". Key Eng. Mater. 297–300, 102–107. <https://doi.org/10.4028/www.scientific.net/KEM.297-300.102>.
- [12] S.M.Barhli, L.Saucedo- Mora, M.S.L.Jordan, A.F.Cinar, C.Reinhard, M.Mostafavi and T.J.Marrow, 2017, "Synchrotron X-ray characterization of crack strain fields in polygranular graphite, Carbon V 124", November, pp. 357-371.
- [13] M. Bentahar, H. Benzaama and N. Mahmoudi, 2021, "Numerical Modeling of the Evolution of the Strain energy ALLSE of the Crack Propagation by The X-FEM Method", Revue des Matériaux et Energies Renouvelable, 15 (2), pp.24-31. <https://www.asjp.cerist.dz/en/article/167392>.
- [14] F. Saverio, 2014, "Modélisation tridimensionnelle de la fermeture induite par plasticité lors de la propagation d'une fissure de fatigue dans l'acier 304L thèse de doctorat", l'école nationale supérieure de mécanique et d'aérotechnique.
- [15] J.R. Rice, (1968). "A path independent integral and the approximate analysis of strain concentrations by notches and cracks". J. of Appl. Mech, 35, pp.379-386.
- [16] H.D. Bui, 1973, "Dualité entre les intégrales de contour". Comptes Rendus Acad. Sciences, T. 276, Paris.
- [17] Nguyen, Q.S., 1980, Méthodes énergétiques en mécanique de la rupture. J. de Méca, 19(2), pp. 363-386.
- [18] Ph. Destuynder, and M. Djaoua, 1981, "Sur une interprétation mathématique de l'intégrale de Rice en théorie de la rupture fragile", Math. Meth. In the Appl. Sci, 3, pp. 70-87.
- [19] H.P. Tada, P.C. Paris, and G.R. Irwin, 2000, "The Stress Analysis of Cracks Handbook", American Society of Mechanical Engineering.
- [20] M. Bentahar, 2023, "ALLDMD Dissipation Energy Analysis by the Method Extended Finite Elements of a 2D Cracked Structure of an Elastic Linear Isotropic Homogeneous Material", Journal of Electronics, Computer Networking and Applied Mathematics, Vol. 03, No. 02, pp. 1-8, DOI: <https://doi.org/10.55529/jecnam.32.1.8>.
- [21] M. Bentahar, 2023, "Fatigue Analysis of an Inclined Crack Propagation Problem by the X-FEM Method", International Journal of Applied and Structural mechanics, Vol. 03, No. 04, June-July, pp. 23-31, <https://doi.org/10.55529/ijasm.34.23.31>.